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# Sharp and Diffuse Fronts in Oil Reservoirs: Front Tracking and Capillarity

J. Glimm, B. Lindquist, O. McBryan  
and G. Tryggvason

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Sharp and Diffuse Fronts in Oil Reservoirs:  
Front Tracking and Capillarity

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ABSTRACT

A fingering interface instability occurs in laboratory studies of flow oil reservoirs. The phenomenon has been captured numerically in a series of papers by the authors and co-workers. The solution of a 20 year old problem posed by Rachford is the lastest in this line of work, and will be discussed here.

Multiple well field scale reservoir studies are presented here using the front tracking method. Since these field scale problems are normally plagued with grids which are coarse in comparison to the details of wells, fronts and geological layers, the potential advantage for the front tracking method, with an increased resolution is considerable for this problem.

The solution of a major problem in the front tracking method was required to obtain the multi-well studies presented here. These studies involve complex and convoluted interfaces due to the merger and pinch off of colliding interfaces. These changes in interface topology are accomplished with the aid of an untangling algorithm and with a knowledge of the relevant elementary waves and Riemann problem solutions in two dimensions, also discussed here. The conclusion is that front tracking can handle messy practical problems much more complex than those previously studied by the authors, which focused on scientific benchmarks in idealized situations where comparison solutions were known.

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1. Introduction. Reservoir computations are used to model laboratory experiments, for detailed studies near a single well and for field scale multi-well simulations. One of the well-known difficulties with these computations is the wealth of detail in the form of wells, fronts and geological layers, which exceeds the resolution of practical grids. Other difficulties arise from the singularities associated with wells, fronts, and layers, and from the non-linear structure of the flow equations.

Front tracking has promise for most of these issues, since it allows increased frontal resolution by the exact representation of discontinuities and utilization of analytic knowledge concerning nonlinear wave interactions. Moreover, it has solved the much discussed problems of numerical diffusion and grid orientation [1].

We consider the water-oil (immiscible) two phase displacement process in two space dimensions described by the equations

$$\vec{v} = \lambda(s) \nabla P \quad (\text{Darcy's Law}) \quad (1.1)$$

$$\nabla \cdot \vec{v} = 0 \quad (\text{incompressibility}) \quad (1.2)$$

$$\frac{s}{t} + \nabla \cdot \vec{v} f(s) = \nabla \cdot (f(s) \nabla s) \quad (\text{saturation equation}) \quad (1.3)$$

The right hand side of the saturation equation is a nonlinear diffusion term associated with capillary pressure. It gives rise to a length scale which is significant for laboratory studies and is usually much smaller than typical grid spacings for field scale simulations. In front tracking computations, this term is (presently) set to zero. As a test of correctness of this procedure, we summarize in the next section the results of King, Lindquist and Reyna [2] which show the same fingering phenomena for the full equations (1.1-1.3) including capillary diffusion, as were reported earlier [3,4] using front tracking with capillary diffusion set to zero. The computations in Ref. 2 are more conventional, being based on finite differences. It would be interesting to carry further the comparison of these two methods, especially in a parameter range where phenomena such as Taylor diffusion may be important. In such regimes, a phenomenological effective or renormalized diffusion length scale may be larger than the bare length scale defined directly by the equations (1.1-1.3).

The main new scientific issue in the application of front tracking to large scale multi-well problems is the problem of change in front topology due to overlapping or crossing fronts. This problem has two aspects. One is mathematical, and has to do with wave interactions and Riemann problems for nonlinear conservation laws in two space dimensions. This problem has been addressed by various authors [5,6,7,8]. Ref. 8 presents the two dimensional Riemann problem solution for two phase incompressible flow in porous media which solves the problem of interacting oil-water interfaces. The other aspect of this problem is the untangling algorithm itself. This algorithm will be discussed in detail in a future paper; here we present an algorithm used for

a related but slightly simpler situation, namely resolving interface tangles which arise in the course of the elliptic grid construction. The design of these algorithms is fairly general and they should be suitable for intersections of scalar waves in nonlinear hyperbolic conservation laws, including contacts and material interfaces in gas dynamics.

Finally we observe that the study of Riemann problems is an important frontier in the theory of hyperbolic conservation laws: data in the large, in two and three dimensions and nonlocal data are all of interest. This study requires the solution to the problem of interaction of nonlinear waves, and so its importance goes well beyond its use in front tracking or higher order Godunov schemes. We mention work in progress [9,10] concerning Riemann problems for three phase oil reservoirs.

Front tracking has passed a range of scientific tests and benchmarks. The major conclusion of this paper is that it has developed to the point where more practical tests closer to engineering practice are possible.

2. Capillarity. The capillary pressure term in (1.3) is relevant on length scales in the range of 6 inches to 30 feet at typical reservoir flow rates. The grid spacings used in field scale calculations lie typically in a range of 100 to 1000 feet. On such grids, most numerical methods yield answers dominated by non-physical diffusion effects, whose length scales are typically at least an order of magnitude larger than the capillary length scales mentioned above. It is not possible to resolve capillary pressure effects correctly on such grids. For problems where these subgrid capillary effects do not modify the large scale behavior of the flow, it is appropriate to set the capillary pressure term to zero. In this approximation, front tracking is exact.

There are two cases in which the possibility of subgrid capillary effects modifying the large scale flow require further analysis. The first case is the possibility, in the unstable mobility ratio regime, that small scale fingers will produce an effective diffusion layer. One version of this scenario is known as Taylor diffusion. We note that experimental observations under laboratory conditions show fingers uninhibited by such diffusion phenomena. Also the numerical computations of Ref. 2, described in more detail below, show a fingering phenomena very similar to that seen under front tracking, suggesting a range of numerical convergence, as capillarity is reduced, to the front tracking solution. Further study of these questions would be desirable.

The other regime requiring further analysis is that in which there is significant internal structure to a front. A flame front is an example of this situation. Front tracking is not applicable to such situations unless it is modified to contain the results of a modeling

analysis which provides adequate representation of the internal front structure. Otherwise locally (or globally) fine grids are required.

With respect to laboratory scale studies, we feel that front tracking may still be adapted to a regime where capillary length scales are on the order of magnitude of a computational grid block. With the intention of finding a way of modeling capillarity reasonably using front tracking we first undertook an exact study of flow with capillarity. Over the past 20 years the results of numerical studies of viscous fingering for immiscible flow have been in disagreement both with laboratory core flood studies and theoretical analysis. In 1964 Rachford [11] studied the linearized behaviour of two phase, incompressible, immiscible viscous flow. He found analytic evidence for the existence of unstable regimes but could produce only weak numerical perturbation growth, far weaker than that found in core flood studies. To the authors' knowledge, no comprehensive study resolving this discrepancy has appeared in the literature, other than Ref. 2.

Rachford's problem has been solved in a two dimensional numerical study of viscous fingering flow under core sample conditions. This study used a finite difference scheme and revealed both the growth of initial perturbations in a homogeneous medium, and fingered flow from an initial one dimensional flow using a heterogeneous medium to initiate the perturbations. In the set of calculations involving a homogeneous medium the dependence of stability on capillary strength, mobility ratio, and initial perturbation size was studied. A large region of phase space in these variables was found to produce unstable growth of initial perturbations. The unstable growth rates were found to be, to a good approximation, linear. The calculations involving a heterogeneous medium investigated the production of unstable growth as a function of the length scale over which the heterogeneity varied. This idea does not seem to have been pursued in the literature and appears to be the key (outside of using a well controlled numerical scheme) to an understanding of the production of unstable growth in heterogeneous medium. The results of our study show that understanding of the interplay of length scales present in the numerical study is essential for a correct numerical calculation of the stability of viscous fingering. Specifically it was found that for instability to occur

- a. The destabilizing length scales (i.e. heterogeneity) must be large relative to the stabilizing length scales (capillarity). All perturbations induced by heterogeneity which are smaller than the scale set by capillary forces are stabilized and do not grow.
- b. These physical length scales must also be larger than the numerical length scales (the grid spacing, and the fiducial length over which the dispersion relation of the truncated difference scheme agrees reasonably with the dispersion relation for the differential equation).

An example of an unstable flow in a heterogeneous medium is shown in Fig. 1. The initial conditions were that of an unperturbed, one

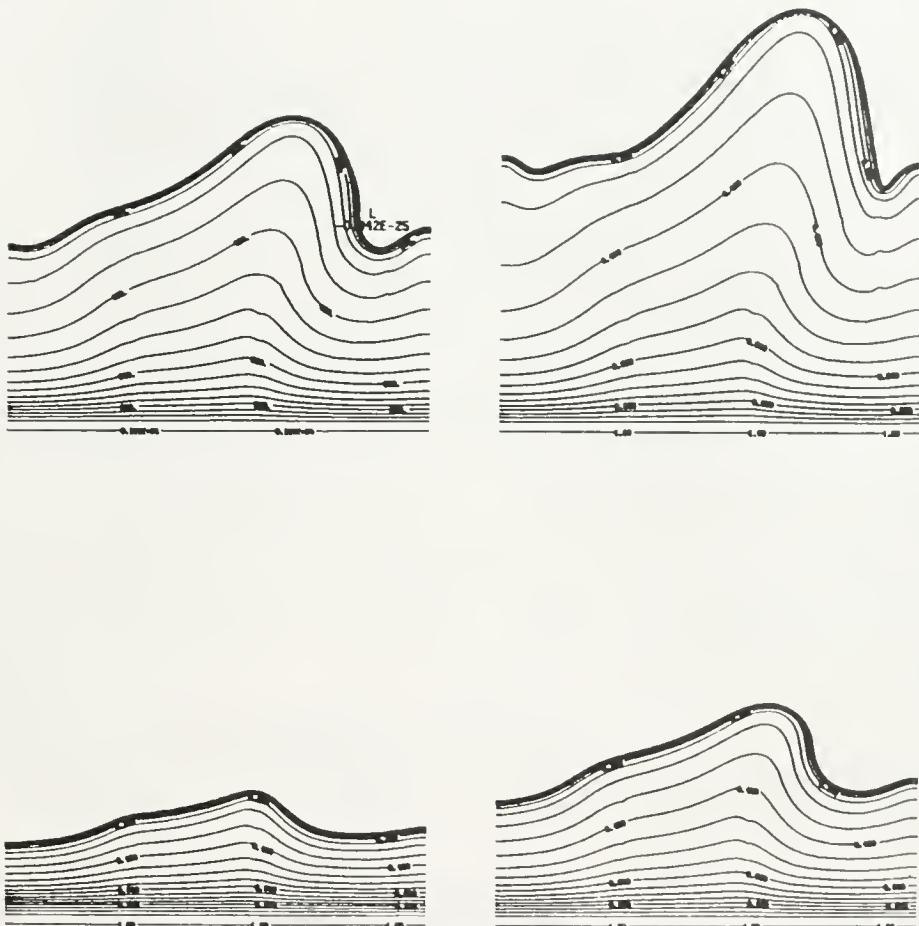


Figure 1. Fingered viscous flow produced in a medium of variable permeability. Shown are the contours of saturation for four equally spaced times in the flow.

dimensional flow. The permeability was varied according to a log-normal distributed random number over a predetermined length scale. In this example the length scale of variation is 1/3 of the computational length. Globally the permeability varied by a factor of four.

3. The Unravel Algorithm. The solution of the elliptic pressure equation (1.1), (1.2) on a given rectangular  $N \times M$  grid, involves the construction of an interface and a division of the grid into elements consisting of rectangles and triangles [12]. The interface obtained at time  $n \Delta t$  in the solution of the saturation equation is used as input to this construction. The grid division is achieved by shifting points of the rectangular grid along grid lines to the nearest point of the input interface, thereby creating irregular quadrilaterals along the interface. This shift is never more than half a mesh spacing. The input interface is now modified and defined as passing only through the shifted node points. The irregular quadrilaterals are now bisected into two triangles each in such a manner that the interface always lies along own side of a triangle. If the input interface from the saturation is too complex and has too many nearby interface curves this algorithm may fail and produce new intersections in the resultant modified interface. A procedure of resolving these intersections has been developed. A generic local tangle involving two curve segments that can be produced and the steps of the untangling procedure are depicted in Fig. 2a. A special case showing an isolated point of intersection is shown in Fig. 2b. The untangling algorithm proceeds as follows:

- Curves are split at points of intersection (open circles, middle figure of Fig. 2a). If two curves were originally parallel over a length defined by several contiguous points of intersection, this splitting procedure produces a series of short, pairwise identical curves.
- All pairs of curves are inspected; if two curves are found to be identical, they are replaced by a single curve.

The untangling algorithm assumes that no more than two curve segments will interact locally after shifting the grid points. If this assumption is violated, the size of the starting  $M \times N$  rectangular grid is increased until the above condition is met.

4. Well Competition. One of the common field techniques in secondary recovery is the use of a line of wells as secondary recovery fluid injection sites. Commonly a line of wells near the reservoir boundaries are chosen. The question of the stability of this 'line drive' arrangement has been addressed via the front tracking method. A series of runs were performed with a line of five water injectors on one boundary of an idealized reservoir and a line of five producers on the opposite. The reservoir is assumed homogeneous. The middle injector is displaced slightly from the line formed by the other four in a favorable direction towards the producers. The runs were performed at various frontal mobility ratios and aspect ratios. Here

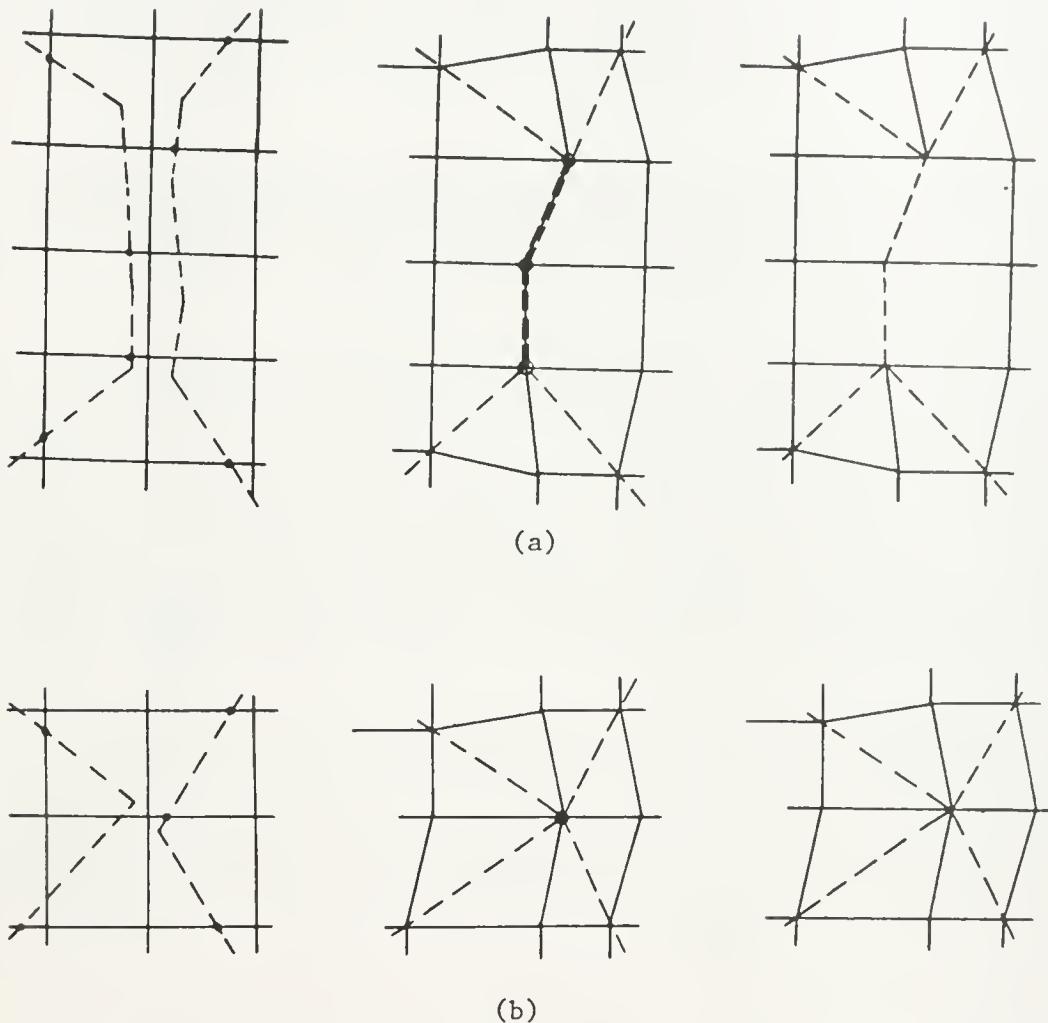


Figure 2. a) A generic tangle involving two curve segments that can be produced by the grid algorithm for the elliptic equation solution (1.1), (1.2). The curves are represented by dotted lines, the grid by solid lines. The situation before shifting grid nodes is shown on the left hand side. The dark points indicate nearest interface points to which the grid nodes will be shifted. The middle figure depicts the resultant shifted grid and modified curves completing the first step of the untangling algorithm described in the text. The open circles represent endpoints of the resultant curves. The figure on the right represents the final untangled interface after the second step of the algorithm. Some grid lines are suppressed for clarity.

b) A special case of a) showing the untangling procedure at an isolated intersection point.

the aspect ratio is the ratio of the distance between the wells in a line to the distance between the two lines of wells. This study was motivated by a suggestion from H. Hanche-Olsen and T. Johansen, Institute of Energy Technology, Oslo, and an earlier study done on a related, but simpler, system [13].

We present two of our calculations, illustrating the dependence of the solution on the degree of interface instability, for two distinct regions of the parameter range. The figures depict the fluid interface at a time value late in each calculation.

In Fig. 3a, consider the case of water injection into a field of oil of equal viscosity to the injected fluid. This calculation is motivated by a field study in Nigeria [14] where a line drive water flood was employed. The leading edge of the injection fluid is highly stable (far field mobility ratio 1.0, frontal mobility ratio 0.586). The separate, initially circular leading edges from each injector, tend to merge to produce a single sweeping front. However, the trailing interface is unstable resulting in fingers which pinch off, producing bubbles of trapped oil. Since the oil is the more mobile phase, these bubbles move through the less mobile water phase and eventually break through the stable leading edge.

This computation was tested under mesh refinement by a factor of two in the x and y directions. It was found that the qualitative behavior in terms of fingers and pinch off was the same; the front calculation shown in Fig. 3a having satisfactorily converged everywhere except in the regions of meandering finger boundaries and pinch off. There the qualitative details were the same but in the finer grid run the meandering and bubble formation occurs on a finer scale than that shown in Fig. 3a.

The data was also analysed in color plots showing saturation levels. The narrow meandering oil fingers, which pinch off and form bubbles lead to alternate flows of water displacing oil and oil displacing water in the regions between the main water fingers. The saturation plots reveal that this phenomena leads to a band where saturation takes on an intermediate value, near the inflection point of the fractional flow curve. In this sense the tracked fronts of the narrow oil fingers represent only a small jump in the saturations, so that the bubbles, as seen in Fig. 3a are not pure oil, but have a water saturation in the range  $s \leq 0.5$ , compared to the forward flood shock height saturation  $s = 0.707$ .

In Fig. 3b, we consider the more common case of a field of more viscous oil. The calculation shown is for a far field mobility ratio of 8 (frontal mobility ratio 1.333) which is representative of a range of more common situations. Here, the displaced center well develops into a favored central finger, with a mushroom cap we have found to be characteristic to unstable injection regimes for these

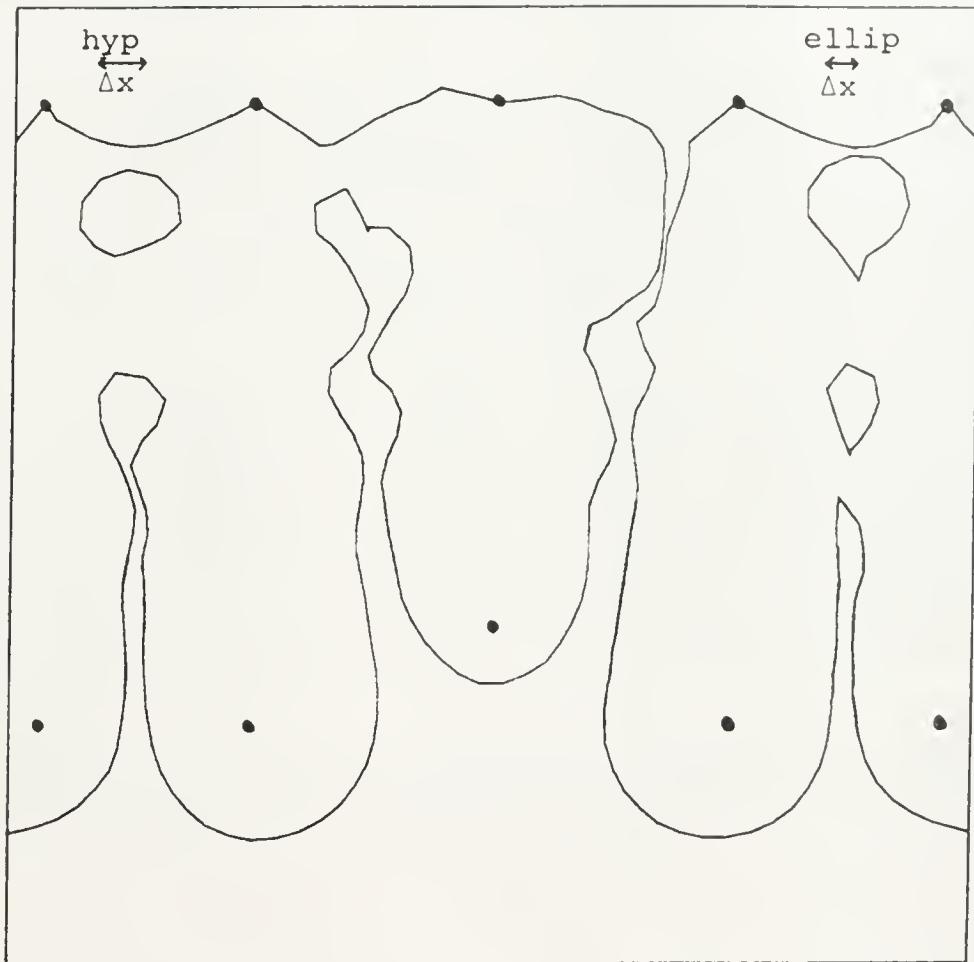


Figure 3a. Line drive displacement with a favored center injection well. The frontal mobility ratio is 0.586 for water displacing oil. Injection and production wells are marked  $\bullet$ . The x axis grid spacings for the elliptic calculation (1.1) (1.2) and for the off-front hyperbolic calculation (1.3) are indicated respectively in the upper right and left corners of the figure.

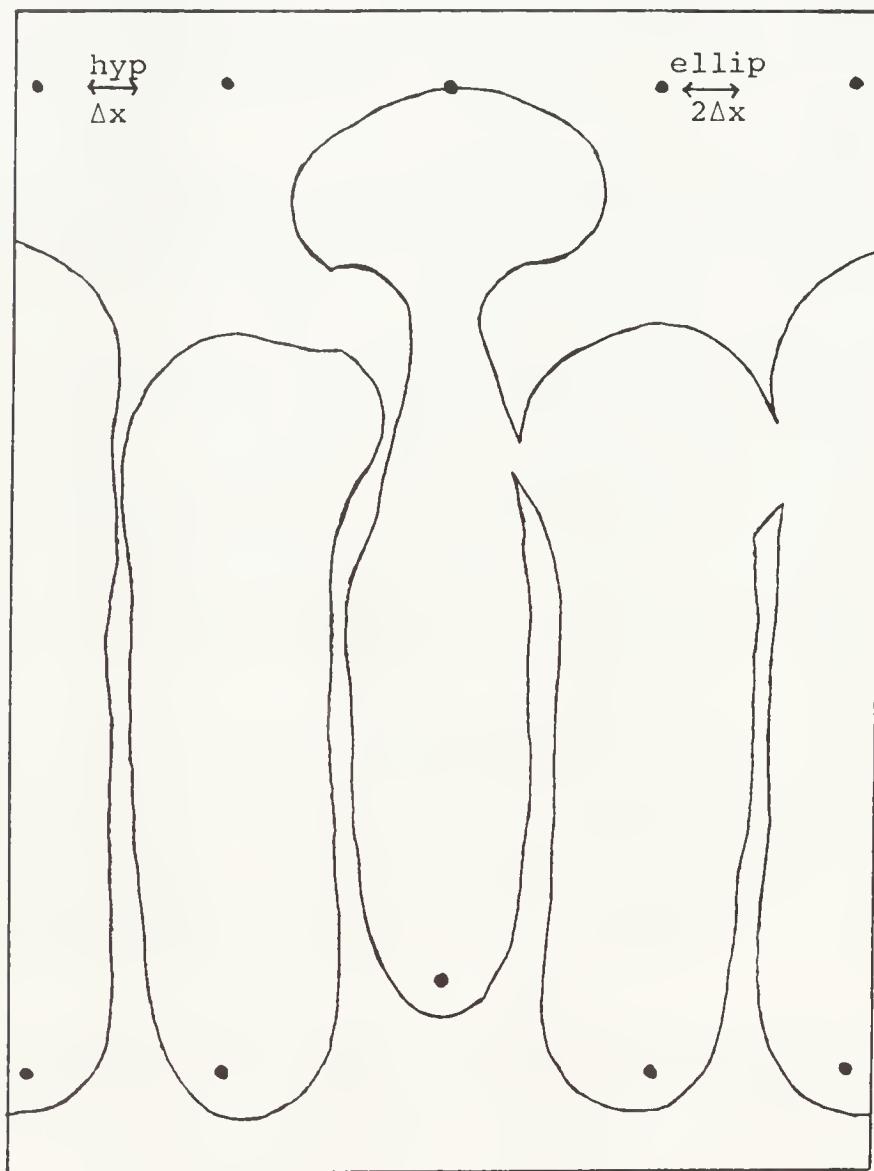


Figure 3b. As in figure 3a but now the displaced fluid is more viscous. The frontal mobility ratio is 1.333. The flow from the favored injector develops a characteristic mushroom shaped instability in the fluid interface. This finger acts as an attractor for the flow of the neighboring wells.

geometries. This finger acts as an attractor for its neighbours.

At still higher mobility ratios, calculations have shown that the trends in Fig. 3b become enhanced, leading to pinch off of the central finger, producing a large mobile water bubble.

5. Multi-Wall Calculations. As an indication of the potential of the front tracking method on larger reservoir scales and to develop the algorithm capabilities towards engineering problems we present two multiple-well calculations. These studies are preliminary in the sense that the interface untangling algorithm used in the calculations is still under development. In Fig. 4a, we show a plot from the calculation of a field having three injection wells (squares with embedded +) and three production wells (open squares). The frontal mobility ratio is 1.4 (far field mobility ratio 10). Shown are the fluid interface at a late time in the calculation (two of the production wells have broken through) together with saturation contours depicting the mixing zone behind the interface. Some small mixing occurs ahead of the front due to earlier interaction amongst the fluid interface before merging. These are identified as mixing layers from regions of backflow due to injection well interactions. Some of this backflow is due to the coarseness of grid for the elliptic solution (32 by 32) and disappears under mesh refinement of the elliptic equation. The grid used for the off-front hyperbolic calculations (see [4]) was 20 by 20. The size of the two grid spacings are indicated on Fig. 4a,

In Fig. 4b, we present an intermediate time value for a run with 8 injection wells and 13 production wells. The frontal mobility ratio is again 1.4 (far field mobility ratio 10). The comments for the previous calculation apply here as well.

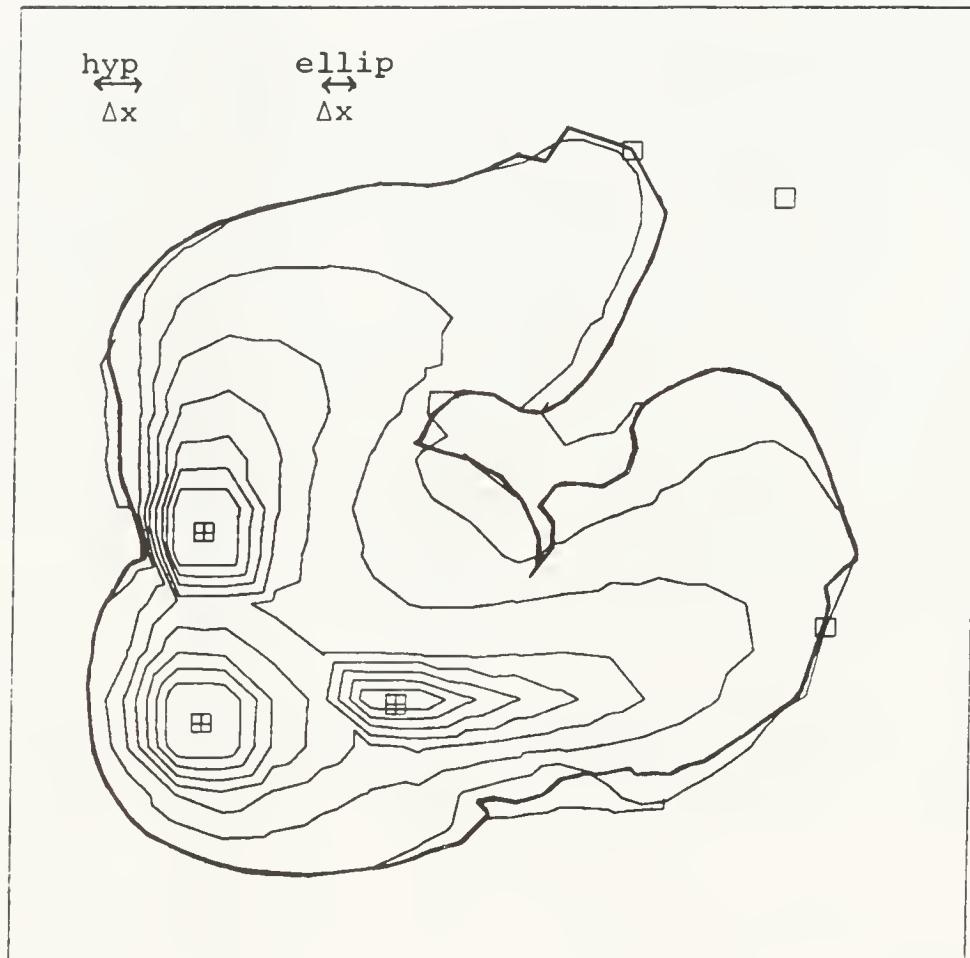


Figure 4a. The fluid interface (dark line) and saturation contours for a time late in the calculation of flow in a six well field. The frontal mobility ratio is 1.4 (far field mobility ratio 10). Injection wells are marked as squares with embedded +, production wells as open squares. The x axis grid spacings for the elliptic calculation (1.1) (1.2) and for the off-front hyperbolic calculation (1.3) are also shown.

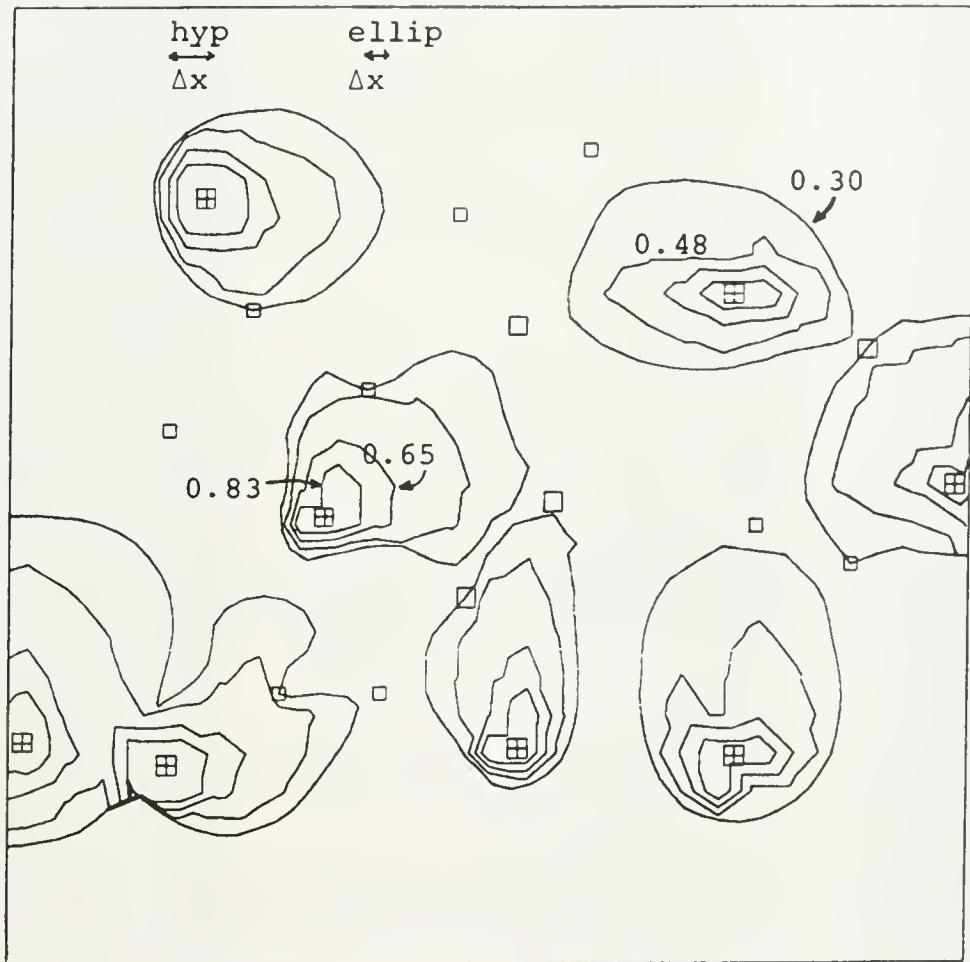


Figure 4b. The fluid interface and saturation contours for a time late in the calculation of flow in a 21 well field. The frontal mobility ratio is 1.4 (far field mobility ratio 10). Injection wells are marked as squares with embedded +, production wells as open squares. The x axis grid spacings for the elliptic calculation (1.1) (1.2) and for the off-front hyperbolic calculation (1.3) are also shown. Four saturation contours are shown around each well. The values of the contour lines are (from the injection well outward) 0.83, 0.65, 0.48, 0.30. The oil water interface occurs at the 0.30 contour.

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